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Die qualitativen Gruppen der 360 Regeln semiotischer CAs

1. Auf der Grundlage der umfassenden Darstellung einer polykontexturalen Semiotik in Toth (2019a) wurden in Toth (2019b-g) die ersten Grundlagen zu einer zukünftigen Theorie semiotischer zellulärer Automaten (CAs) geschaffen. Semiotische CAs sind ihrer Natur gemäß qualitativ, denn das Zeichen ist nach Peirce eine triadische Relation, die nicht nur über einen Mittelbezug, sondern auch über einen Objekt- und einen Interpretantenbezug verfügt, d.h. das Zeichen, wie es von Bense (1981, S. 17 ff.) als Zeichenzahl eingeführt wurde, rechnet auch mit Sinn und Bedeutung. Wie im folgenden gezeigt wird, korrespondieren den $2^8 = 256$ Regeln der quantitativen CAs (vgl. Gardner 1970) 360 Regeln der qualitativen (semiotischen) CAs. Diese lassen sich, wie ebenfalls gezeigt wird, in Paare gleicher Outputs für jede der 15 Gruppen zu je 24 CAs subgruppieren.

2. Kategorietheoretische Notation der 360 Regeln semiotischer CAs

2.1. R = (0, 00, 01, 000)

$$(\gamma \rightarrow \beta^\circ \gamma^\circ) \rightarrow \alpha^\circ \beta^\circ \gamma^\circ$$

$$(\gamma \beta \rightarrow \gamma^\circ) \rightarrow \alpha^\circ \beta^\circ \gamma^\circ$$

$$(\beta \rightarrow \gamma) \rightarrow \alpha^\circ \beta^\circ$$

$$(\gamma^\circ \rightarrow \beta^\circ) \rightarrow \alpha^\circ \beta^\circ$$

$$(\gamma \beta \alpha \rightarrow \gamma^\circ) \rightarrow \beta^\circ \gamma^\circ$$

$$(\gamma \rightarrow \alpha^\circ \beta^\circ \gamma^\circ) \rightarrow \beta^\circ \gamma^\circ$$

$$(\beta \alpha \rightarrow \gamma) \rightarrow \beta^\circ$$

$$(\gamma^\circ \rightarrow \alpha^\circ \beta^\circ) \rightarrow \beta^\circ$$

$$(\beta^\circ \rightarrow \gamma \beta) \rightarrow \alpha^\circ$$

$$(\beta^\circ \gamma^\circ \rightarrow \beta) \rightarrow \alpha^\circ$$

$$(\alpha^\circ \rightarrow \gamma \beta \alpha) \rightarrow \beta \alpha$$

$$(\alpha^\circ \beta^\circ \gamma^\circ \rightarrow \alpha) \rightarrow \beta \alpha$$

$$(\alpha \rightarrow \gamma\beta) \rightarrow \beta$$
$$(\beta^\circ\gamma^\circ \rightarrow \alpha^\circ) \rightarrow \beta$$
$$(\alpha^\circ\beta^\circ \rightarrow \gamma\beta\alpha) \rightarrow \alpha$$
$$(\alpha^\circ\beta^\circ\gamma^\circ \rightarrow \beta\alpha) \rightarrow \alpha$$
$$(\alpha^\circ \rightarrow \beta\alpha) \rightarrow \gamma\beta\alpha$$
$$(\alpha^\circ\beta^\circ \rightarrow \alpha) \rightarrow \gamma\beta\alpha$$
$$(\alpha \rightarrow \beta) \rightarrow \gamma\beta$$
$$(\beta^\circ \rightarrow \alpha^\circ) \rightarrow \gamma\beta$$
$$(\gamma\beta\alpha \rightarrow \beta^\circ\gamma^\circ) \rightarrow \gamma^\circ$$
$$(\gamma\beta \rightarrow \alpha^\circ\beta^\circ\gamma^\circ) \rightarrow \gamma^\circ$$
$$(\beta\alpha \rightarrow \beta^\circ) \rightarrow \gamma$$
$$(\beta \rightarrow \alpha^\circ\beta^\circ) \rightarrow \gamma$$

2.2. R = (0, 00, 01, 001)

$$(\delta\gamma\beta \rightarrow \gamma^\circ\delta^\circ) \rightarrow \alpha^\circ\beta^\circ\gamma^\circ\delta^\circ$$
$$(\delta\gamma \rightarrow \beta^\circ\gamma^\circ\delta^\circ) \rightarrow \alpha^\circ\beta^\circ\gamma^\circ\delta^\circ$$
$$(\beta \rightarrow \delta\gamma) \rightarrow \alpha^\circ\beta^\circ$$
$$(\gamma^\circ\delta^\circ \rightarrow \beta^\circ) \rightarrow \alpha^\circ\beta^\circ$$
$$(\beta^\circ \rightarrow \delta\gamma\beta) \rightarrow \alpha^\circ$$
$$(\beta^\circ\gamma^\circ\delta^\circ \rightarrow \beta) \rightarrow \alpha^\circ$$
$$(\delta\gamma\beta\alpha \rightarrow \gamma^\circ\delta^\circ) \rightarrow \beta^\circ\gamma^\circ\delta^\circ$$
$$(\delta\gamma \rightarrow \alpha^\circ\beta^\circ\gamma^\circ\delta^\circ) \rightarrow \beta^\circ\gamma^\circ\delta^\circ$$
$$(\beta\alpha \rightarrow \delta\gamma) \rightarrow \beta^\circ$$
$$(\gamma^\circ\delta^\circ \rightarrow \alpha^\circ\beta^\circ) \rightarrow \beta^\circ$$
$$(\delta\gamma\beta\alpha \rightarrow \beta^\circ\gamma^\circ\alpha^\circ) \rightarrow \gamma^\circ\delta^\circ$$

$$(\delta\gamma\beta \rightarrow \alpha^\circ\beta^\circ\gamma^\circ\delta^\circ) \rightarrow \gamma^\circ\delta^\circ$$

$$(\alpha^\circ \rightarrow \delta\gamma\beta\alpha) \rightarrow \beta\alpha$$

$$(\alpha^\circ\beta^\circ\gamma^\circ\delta^\circ \rightarrow \alpha) \rightarrow \beta\alpha$$

$$(\alpha \rightarrow \delta\gamma\beta) \rightarrow \beta$$

$$(\beta^\circ\gamma^\circ\alpha^\circ \rightarrow \alpha^\circ) \rightarrow \beta$$

$$(\alpha^\circ\beta^\circ \rightarrow \delta\gamma\beta\alpha) \rightarrow \alpha$$

$$(\alpha^\circ\beta^\circ\gamma^\circ\delta^\circ \rightarrow \beta\alpha) \rightarrow \alpha$$

$$(\alpha^\circ \rightarrow \beta\alpha) \rightarrow \delta\gamma\beta\alpha$$

$$(\alpha^\circ\beta^\circ \rightarrow \alpha) \rightarrow \delta\gamma\beta\alpha$$

$$(\alpha \rightarrow \beta) \rightarrow \delta\gamma\beta$$

$$(\beta^\circ \rightarrow \alpha^\circ) \rightarrow \delta\gamma\beta$$

$$(\beta\alpha \rightarrow \beta^\circ) \rightarrow \delta\gamma$$

$$(\beta \rightarrow \alpha^\circ\beta^\circ) \rightarrow \delta\gamma$$

2.3. R = (0, 00, 01, 012)

$$(\varepsilon\delta\gamma\beta \rightarrow \gamma^\circ\delta^\circ\varepsilon^\circ) \rightarrow \alpha^\circ\beta^\circ\gamma^\circ\delta^\circ\varepsilon^\circ$$

$$(\varepsilon\delta\gamma \rightarrow \beta^\circ\gamma^\circ\delta^\circ\varepsilon^\circ) \rightarrow \alpha^\circ\beta^\circ\gamma^\circ\delta^\circ\varepsilon^\circ$$

$$(\varepsilon\delta\gamma\beta\alpha \rightarrow \gamma^\circ\delta^\circ\varepsilon^\circ) \rightarrow \beta^\circ\gamma^\circ\delta^\circ\varepsilon^\circ$$

$$(\varepsilon\delta\gamma \rightarrow \alpha^\circ\beta^\circ\gamma^\circ\delta^\circ\varepsilon^\circ) \rightarrow \beta^\circ\gamma^\circ\delta^\circ\varepsilon^\circ$$

$$(\varepsilon\delta\gamma\beta\alpha \rightarrow \beta^\circ\gamma^\circ\delta^\circ\varepsilon^\circ) \rightarrow \gamma^\circ\delta^\circ\varepsilon^\circ$$

$$(\varepsilon\delta\gamma\beta \rightarrow \alpha^\circ\beta^\circ\gamma^\circ\delta^\circ\varepsilon^\circ) \rightarrow \gamma^\circ\delta^\circ\varepsilon^\circ$$

$$(\beta \rightarrow \varepsilon\delta\gamma) \rightarrow \alpha^\circ\beta^\circ$$

$$(\gamma^\circ\delta^\circ\varepsilon^\circ \rightarrow \beta^\circ) \rightarrow \alpha^\circ\beta^\circ$$

$$(\beta\alpha \rightarrow \varepsilon\delta\gamma) \rightarrow \beta^\circ$$

$$(\gamma^\circ\delta^\circ\varepsilon^\circ \rightarrow \alpha^\circ\beta^\circ) \rightarrow \beta^\circ$$

$(\beta^\circ \rightarrow \varepsilon\delta\gamma\beta) \rightarrow \alpha^\circ$
 $(\alpha^\circ\beta^\circ\gamma^\circ\delta^\circ\varepsilon^\circ \rightarrow \beta) \rightarrow \alpha^\circ$
 $(\alpha^\circ \rightarrow \varepsilon\delta\gamma\alpha) \rightarrow \beta\alpha$
 $(\alpha^\circ\beta^\circ\gamma^\circ\delta^\circ\varepsilon^\circ \rightarrow \alpha) \rightarrow \beta\alpha$
 $(\alpha \rightarrow \varepsilon\delta\gamma\beta) \rightarrow \beta$
 $(\beta^\circ\gamma^\circ\delta^\circ\varepsilon^\circ \rightarrow \alpha^\circ) \rightarrow \beta$
 $(\alpha^\circ\beta^\circ \rightarrow \varepsilon\delta\gamma\beta\alpha) \rightarrow \alpha$
 $(\alpha^\circ\beta^\circ\gamma^\circ\delta^\circ\varepsilon^\circ \rightarrow \beta\alpha) \rightarrow \alpha$
 $(\alpha^\circ \rightarrow \beta\alpha) \rightarrow \varepsilon\delta\gamma\beta\alpha$
 $(\alpha^\circ\beta^\circ \rightarrow \alpha) \rightarrow \varepsilon\delta\gamma\beta\alpha$
 $(\alpha \rightarrow \beta) \rightarrow \varepsilon\delta\gamma\beta$
 $(\beta^\circ \rightarrow \alpha^\circ) \rightarrow \varepsilon\delta\gamma\beta$
 $(\beta\alpha \rightarrow \beta^\circ) \rightarrow \varepsilon\delta\gamma$
 $(\beta \rightarrow \alpha^\circ\beta^\circ) \rightarrow \varepsilon\delta\gamma$

2.4. $R = (0, 00, 000, 001)$

$(\delta\gamma\beta \rightarrow \delta^\circ) \rightarrow \alpha^\circ\beta^\circ\gamma^\circ\delta^\circ$
 $(\delta \rightarrow \beta^\circ\gamma^\circ\delta^\circ) \rightarrow \alpha^\circ\beta^\circ\gamma^\circ\delta^\circ$
 $(\gamma\beta \rightarrow \delta) \rightarrow \alpha^\circ\beta^\circ\gamma^\circ$
 $(\delta^\circ \rightarrow \beta^\circ\gamma^\circ) \rightarrow \alpha^\circ\beta^\circ\gamma^\circ$
 $(\delta\gamma\beta\alpha \rightarrow \delta^\circ) \rightarrow \beta^\circ\gamma^\circ\delta^\circ$
 $(\delta \rightarrow \alpha^\circ\beta^\circ\gamma^\circ\delta^\circ) \rightarrow \beta^\circ\gamma^\circ\delta^\circ$
 $(\gamma\beta\alpha \rightarrow \delta) \rightarrow \beta^\circ\gamma^\circ$
 $(\delta^\circ \rightarrow \alpha^\circ\beta^\circ\gamma^\circ) \rightarrow \beta^\circ\gamma^\circ$
 $(\beta^\circ\gamma^\circ \rightarrow \delta\gamma\beta) \rightarrow \alpha^\circ$

$(\beta^\circ \gamma^\circ \delta^\circ \rightarrow \gamma \beta) \rightarrow \alpha^\circ$
 $(\alpha^\circ \beta^\circ \gamma^\circ \rightarrow \delta \gamma \beta \alpha) \rightarrow \alpha$
 $(\alpha^\circ \beta^\circ \gamma^\circ \delta^\circ \rightarrow \gamma \beta \alpha) \rightarrow \alpha$
 $(\alpha^\circ \rightarrow \gamma \beta \alpha) \rightarrow \delta \gamma \beta \alpha$
 $(\alpha^\circ \beta^\circ \gamma^\circ \rightarrow \alpha) \rightarrow \delta \gamma \beta \alpha$
 $(\alpha \rightarrow \gamma \beta) \rightarrow \delta \gamma \beta$
 $(\beta^\circ \gamma^\circ \rightarrow \alpha^\circ) \rightarrow \delta \gamma \beta$
 $(\alpha^\circ \rightarrow \delta \gamma \beta \alpha) \rightarrow \gamma \beta \alpha$
 $(\alpha^\circ \beta^\circ \gamma^\circ \delta^\circ \rightarrow \alpha) \rightarrow \gamma \beta \alpha$
 $(\alpha \rightarrow \delta \gamma \beta) \rightarrow \gamma \beta$
 $(\beta^\circ \gamma^\circ \delta^\circ \rightarrow \alpha^\circ) \rightarrow \gamma \beta$
 $(\delta \gamma \beta \alpha \rightarrow \beta^\circ \gamma^\circ \delta^\circ) \rightarrow \delta^\circ$
 $(\delta \gamma \beta \rightarrow \alpha^\circ \beta^\circ \gamma^\circ \delta^\circ) \rightarrow \delta^\circ$
 $(\gamma \beta \alpha \rightarrow \beta^\circ \gamma^\circ) \rightarrow \delta$
 $(\gamma \beta \rightarrow \alpha^\circ \beta^\circ \gamma^\circ) \rightarrow \delta$

2.5. $R = (0, 00, 000, 012)$

$(\varepsilon \delta \gamma \beta \rightarrow \delta^\circ \varepsilon^\circ) \rightarrow \alpha^\circ \beta^\circ \gamma^\circ \delta^\circ \varepsilon^\circ$
 $(\varepsilon \delta \rightarrow \beta^\circ \gamma^\circ \delta^\circ \varepsilon^\circ) \rightarrow \alpha^\circ \beta^\circ \gamma^\circ \delta^\circ \varepsilon^\circ$
 $(\gamma \beta \rightarrow \varepsilon \delta) \rightarrow \alpha^\circ \beta^\circ \gamma^\circ$
 $(\delta^\circ \varepsilon^\circ \rightarrow \beta^\circ \gamma^\circ) \rightarrow \alpha^\circ \beta^\circ \gamma^\circ$
 $(\varepsilon \delta \gamma \beta \alpha \rightarrow \delta^\circ \varepsilon^\circ) \rightarrow \beta^\circ \gamma^\circ \delta^\circ \varepsilon^\circ$
 $(\varepsilon \delta \rightarrow \alpha^\circ \beta^\circ \gamma^\circ \delta^\circ \varepsilon^\circ) \rightarrow \beta^\circ \gamma^\circ \delta^\circ \varepsilon^\circ$
 $(\gamma \beta \alpha \rightarrow \varepsilon \delta) \rightarrow \beta^\circ \gamma^\circ$
 $(\delta^\circ \varepsilon^\circ \rightarrow \alpha^\circ \beta^\circ \gamma^\circ) \rightarrow \beta^\circ \gamma^\circ$

$(\beta^\circ \gamma^\circ \rightarrow \varepsilon \delta \gamma \beta) \rightarrow \alpha^\circ$
 $(\beta^\circ \gamma^\circ \delta^\circ \varepsilon^\circ \rightarrow \gamma \beta) \rightarrow \alpha^\circ$
 $(\alpha^\circ \rightarrow \varepsilon \delta \gamma \beta \alpha) \rightarrow \gamma \beta \alpha$
 $(\alpha^\circ \beta^\circ \gamma^\circ \delta^\circ \varepsilon^\circ \rightarrow \alpha) \rightarrow \gamma \beta \alpha$
 $(\alpha \rightarrow \varepsilon \delta \gamma \beta) \rightarrow \gamma \beta$
 $(\beta^\circ \gamma^\circ \delta^\circ \varepsilon^\circ \rightarrow \alpha^\circ) \rightarrow \gamma \beta$
 $(\alpha^\circ \beta^\circ \gamma^\circ \rightarrow \varepsilon \delta \gamma \beta \alpha) \rightarrow \alpha$
 $(\alpha^\circ \beta^\circ \gamma^\circ \delta^\circ \varepsilon^\circ \rightarrow \gamma \beta \alpha) \rightarrow \alpha$
 $(\varepsilon \delta \gamma \beta \alpha \rightarrow \beta^\circ \gamma^\circ \delta^\circ \varepsilon^\circ) \rightarrow \delta^\circ \varepsilon^\circ$
 $(\varepsilon \delta \gamma \beta \rightarrow \alpha^\circ \beta^\circ \gamma^\circ \delta^\circ \varepsilon^\circ) \rightarrow \delta^\circ \varepsilon^\circ$
 $(\alpha^\circ \rightarrow \gamma \beta \alpha) \rightarrow \varepsilon \delta \gamma \beta \alpha$
 $(\alpha^\circ \beta^\circ \gamma^\circ \rightarrow \alpha) \rightarrow \varepsilon \delta \gamma \beta \alpha$
 $(\alpha, \gamma \beta) \rightarrow \varepsilon \delta \gamma \beta$
 $(\beta^\circ \gamma^\circ \rightarrow \alpha^\circ) \rightarrow \varepsilon \delta \gamma \beta$
 $(\gamma \beta \alpha \rightarrow \beta^\circ \gamma^\circ) \rightarrow \varepsilon \delta$
 $(\gamma \beta \rightarrow \alpha^\circ \beta^\circ \gamma^\circ) \rightarrow \varepsilon \delta$

2.6. $R = (0, 00, 001, 012)$

$(\varepsilon \delta \gamma \beta \rightarrow \varepsilon^\circ) \rightarrow \alpha^\circ \beta^\circ \gamma^\circ \delta^\circ \varepsilon^\circ$
 $(\varepsilon \rightarrow \beta^\circ \gamma^\circ \delta^\circ \varepsilon^\circ) \rightarrow \alpha^\circ \beta^\circ \gamma^\circ \delta^\circ \varepsilon^\circ$
 $(\delta \gamma \beta \rightarrow \varepsilon) \rightarrow \alpha^\circ \beta^\circ \gamma^\circ \delta^\circ$
 $(\varepsilon^\circ \rightarrow \beta^\circ \gamma^\circ \delta^\circ) \rightarrow \alpha^\circ \beta^\circ \gamma^\circ \delta^\circ$
 $(\varepsilon \delta \gamma \beta \alpha \rightarrow \varepsilon^\circ) \rightarrow \beta^\circ \gamma^\circ \delta^\circ \varepsilon^\circ$
 $(\varepsilon \rightarrow \alpha^\circ \beta^\circ \gamma^\circ \delta^\circ \varepsilon^\circ) \rightarrow \beta^\circ \gamma^\circ \delta^\circ \varepsilon^\circ$
 $(\delta \gamma \beta \alpha \rightarrow \varepsilon) \rightarrow \beta^\circ \gamma^\circ \delta^\circ$

$$\begin{aligned}
& (\varepsilon^\circ \rightarrow \alpha^\circ \beta^\circ \gamma^\circ \delta^\circ) \rightarrow \beta^\circ \gamma^\circ \delta^\circ \\
& (\beta^\circ \gamma^\circ \delta^\circ \rightarrow \varepsilon \delta \gamma \beta) \rightarrow \alpha^\circ \\
& (\beta^\circ \gamma^\circ \delta^\circ \varepsilon^\circ \rightarrow \delta \gamma \beta) \rightarrow \alpha^\circ \\
& (\alpha^\circ \beta^\circ \gamma^\circ \delta^\circ \rightarrow \varepsilon \delta \gamma \beta \alpha) \rightarrow \alpha \\
& (\alpha^\circ \beta^\circ \gamma^\circ \delta^\circ \varepsilon^\circ \rightarrow \delta \gamma \beta \alpha) \rightarrow \alpha \\
& (\alpha^\circ \rightarrow \delta \gamma \beta \alpha) \rightarrow \varepsilon \delta \gamma \beta \alpha \\
& (\alpha^\circ \beta^\circ \gamma^\circ \delta^\circ \rightarrow \alpha) \rightarrow \varepsilon \delta \gamma \beta \alpha \\
& (\alpha \rightarrow \delta \gamma \beta) \rightarrow \varepsilon \delta \gamma \beta \\
& (\beta^\circ \gamma^\circ \delta^\circ \rightarrow \alpha^\circ) \rightarrow \varepsilon \delta \gamma \beta \\
& (\alpha^\circ \rightarrow \varepsilon \delta \gamma \beta \alpha) \rightarrow \delta \gamma \beta \alpha \\
& (\alpha^\circ \beta^\circ \gamma^\circ \delta^\circ \varepsilon^\circ \rightarrow \alpha) \rightarrow \delta \gamma \beta \alpha \\
& (\alpha \rightarrow \varepsilon \delta \gamma \beta) \rightarrow \delta \gamma \beta \\
& (\beta^\circ \gamma^\circ \delta^\circ \varepsilon^\circ \rightarrow \alpha^\circ) \rightarrow \delta \gamma \beta \\
& (\varepsilon \delta \gamma \beta \alpha \rightarrow \varepsilon \delta \gamma \beta) \rightarrow \varepsilon^\circ \\
& (\varepsilon \delta \gamma \beta \rightarrow \alpha^\circ \beta^\circ \gamma^\circ \delta^\circ \varepsilon^\circ) \rightarrow \varepsilon^\circ \\
& (\delta \gamma \beta \alpha \rightarrow \beta^\circ \gamma^\circ \delta^\circ) \rightarrow \varepsilon \\
& (\delta \gamma \beta \rightarrow \alpha^\circ \beta^\circ \gamma^\circ \delta^\circ) \rightarrow \varepsilon
\end{aligned}$$

2.7. $R = (0, 01, 000, 001)$

$$\begin{aligned}
& (\delta \gamma \rightarrow \delta^\circ) \rightarrow \alpha^\circ \beta^\circ \gamma^\circ \delta^\circ \\
& (\delta \rightarrow \gamma^\circ \delta^\circ) \rightarrow \alpha^\circ \beta^\circ \gamma^\circ \delta^\circ \\
& (\gamma \rightarrow \delta) \rightarrow \alpha^\circ \beta^\circ \gamma^\circ \\
& (\delta^\circ \rightarrow \gamma^\circ) \rightarrow \alpha^\circ \beta^\circ \gamma^\circ \\
& (\gamma^\circ \rightarrow \delta \gamma) \rightarrow \alpha^\circ \beta^\circ \\
& (\gamma^\circ \delta^\circ \rightarrow \gamma) \rightarrow \alpha^\circ \beta^\circ
\end{aligned}$$

$(\delta\gamma\beta\alpha \rightarrow \delta^\circ) \rightarrow \gamma^\circ\delta^\circ$
 $(\delta \rightarrow \alpha^\circ\beta^\circ\gamma^\circ\delta^\circ) \rightarrow \gamma^\circ\delta^\circ$
 $(\gamma\beta\alpha \rightarrow \delta) \rightarrow \gamma^\circ$
 $(\delta^\circ \rightarrow \alpha^\circ\beta^\circ\gamma^\circ) \rightarrow \gamma^\circ$
 $(\alpha^\circ\beta^\circ \rightarrow \gamma\beta\alpha) \rightarrow \delta\gamma\beta\alpha$
 $(\alpha^\circ\beta^\circ\gamma^\circ \rightarrow \beta\alpha) \rightarrow \delta\gamma\beta\alpha$
 $(\beta\alpha \rightarrow \gamma) \rightarrow \delta\gamma$
 $(\gamma^\circ \rightarrow \alpha^\circ\beta^\circ) \rightarrow \delta\gamma$
 $(\alpha^\circ\beta^\circ\gamma^\circ\delta^\circ \rightarrow \beta\alpha) \rightarrow \gamma\beta\alpha$
 $(\alpha^\circ\beta^\circ \rightarrow \delta\gamma\beta\alpha) \rightarrow \gamma\beta\alpha$
 $(\alpha^\circ\beta^\circ\gamma^\circ \rightarrow \delta\gamma\beta\alpha) \rightarrow \beta\alpha$
 $(\alpha^\circ\beta^\circ\gamma^\circ\delta^\circ \rightarrow \gamma\beta\alpha) \rightarrow \beta\alpha$
 $(\beta\alpha \rightarrow \delta\gamma) \rightarrow \gamma$
 $(\gamma^\circ\delta^\circ \rightarrow \alpha^\circ\beta^\circ) \rightarrow \gamma$
 $(\delta\gamma\beta\alpha \rightarrow \gamma^\circ\delta^\circ) \rightarrow \delta^\circ$
 $(\delta\gamma \rightarrow \alpha^\circ\beta^\circ\gamma^\circ\delta^\circ) \rightarrow \delta^\circ$
 $(\gamma\beta\alpha \rightarrow \gamma^\circ) \rightarrow \delta$
 $(\gamma \rightarrow \alpha^\circ\beta^\circ\gamma^\circ) \rightarrow \delta$

2.8. $R = (0, 01, 000, 012)$

$(\varepsilon\delta\gamma \rightarrow \delta^\circ\varepsilon^\circ) \rightarrow \alpha^\circ\beta^\circ\gamma^\circ\delta^\circ\varepsilon^\circ$
 $(\varepsilon\delta \rightarrow \gamma^\circ\delta^\circ\varepsilon^\circ) \rightarrow \alpha^\circ\beta^\circ\gamma^\circ\delta^\circ\varepsilon^\circ$
 $(\gamma \rightarrow \varepsilon\delta) \rightarrow \alpha^\circ\beta^\circ\gamma^\circ$
 $(\delta^\circ\varepsilon^\circ \rightarrow \gamma^\circ) \rightarrow \alpha^\circ\beta^\circ\gamma^\circ$
 $(\gamma^\circ \rightarrow \varepsilon\delta\gamma) \rightarrow \alpha^\circ\beta^\circ$

$(\gamma^\circ \delta^\circ \varepsilon^\circ \rightarrow \gamma) \rightarrow \alpha^\circ \beta^\circ$
 $(\varepsilon \delta \gamma \beta \alpha \rightarrow \delta^\circ \varepsilon^\circ) \rightarrow \gamma^\circ \delta^\circ \varepsilon$
 $(\varepsilon \delta \rightarrow \alpha^\circ \beta^\circ \gamma^\circ \delta^\circ \varepsilon^\circ) \rightarrow \gamma^\circ \delta^\circ \varepsilon^\circ$
 $(\gamma \beta \alpha \rightarrow \varepsilon \delta) \rightarrow \gamma^\circ$
 $(\delta^\circ \varepsilon^\circ \rightarrow \alpha^\circ \beta^\circ \gamma^\circ) \rightarrow \gamma^\circ$
 $(\alpha^\circ \beta^\circ \rightarrow \varepsilon \delta \gamma \beta \alpha) \rightarrow \gamma \beta \alpha$
 $(\delta^\circ \varepsilon^\circ \rightarrow \beta \alpha) \rightarrow \gamma \beta \alpha$
 $(\alpha^\circ \beta^\circ \gamma^\circ \rightarrow \varepsilon \delta \gamma \beta \alpha) \rightarrow \beta \alpha$
 $(\alpha^\circ \beta^\circ \gamma^\circ \delta^\circ \varepsilon^\circ \rightarrow \gamma \beta \alpha) \rightarrow \beta \alpha$
 $(\beta \alpha \rightarrow \varepsilon \delta \gamma) \rightarrow \gamma$
 $(\gamma^\circ \delta^\circ \varepsilon^\circ \rightarrow \alpha^\circ \beta^\circ) \rightarrow \gamma$
 $(\varepsilon \delta \gamma \beta \alpha \rightarrow \gamma^\circ \delta^\circ \varepsilon^\circ) \rightarrow \delta^\circ \varepsilon^\circ$
 $(\varepsilon \delta \gamma \rightarrow \alpha^\circ \beta^\circ \gamma^\circ \delta^\circ \varepsilon^\circ) \rightarrow \delta^\circ \varepsilon^\circ$
 $(\alpha^\circ \beta^\circ \rightarrow \gamma \beta \alpha) \rightarrow \varepsilon \delta \gamma \beta \alpha$
 $(\alpha^\circ \beta^\circ \gamma^\circ \rightarrow \beta \alpha) \rightarrow \varepsilon \delta \gamma \beta \alpha$
 $(\beta \alpha \rightarrow \gamma) \rightarrow \varepsilon \delta \gamma$
 $(\gamma \rightarrow \alpha^\circ \beta^\circ) \rightarrow \varepsilon \delta \gamma$
 $(\gamma \beta \alpha \rightarrow \gamma^\circ) \rightarrow \varepsilon \delta$
 $(\gamma \rightarrow \alpha^\circ \beta^\circ \gamma^\circ) \rightarrow \varepsilon \delta$

2.9. $R = (0, 01, 001, 012)$

$(\varepsilon \delta \gamma \rightarrow \varepsilon^\circ) \rightarrow \alpha^\circ \beta^\circ \gamma^\circ \delta^\circ \varepsilon^\circ$
 $(\varepsilon \rightarrow \gamma^\circ \delta^\circ \varepsilon^\circ) \rightarrow \alpha^\circ \beta^\circ \gamma^\circ \delta^\circ \varepsilon^\circ$
 $(\delta \gamma \rightarrow \varepsilon) \rightarrow \alpha^\circ \beta^\circ \gamma^\circ \delta^\circ$
 $(\varepsilon^\circ \rightarrow \gamma^\circ \delta^\circ) \rightarrow \alpha^\circ \beta^\circ \gamma^\circ \delta^\circ$

$(\gamma^\circ \delta^\circ \rightarrow \varepsilon \delta \gamma) \rightarrow \alpha^\circ \beta^\circ$
 $(\alpha^\circ \beta^\circ \gamma^\circ \delta^\circ \varepsilon^\circ \rightarrow \delta \gamma) \rightarrow \alpha^\circ \beta^\circ$
 $(\varepsilon \delta \gamma \beta \alpha \rightarrow \varepsilon^\circ) \rightarrow \gamma^\circ \delta^\circ \varepsilon^\circ$
 $(\varepsilon \rightarrow \alpha^\circ \beta^\circ \gamma^\circ \delta^\circ \varepsilon^\circ) \rightarrow \gamma^\circ \delta^\circ \varepsilon^\circ$
 $(\delta \gamma \beta \alpha \rightarrow \varepsilon) \rightarrow \gamma^\circ \delta^\circ$
 $(\varepsilon^\circ \rightarrow \alpha^\circ \beta^\circ \gamma^\circ \delta^\circ) \rightarrow \gamma^\circ \delta^\circ$
 $(\varepsilon \delta \gamma \beta \alpha \rightarrow \gamma^\circ \delta^\circ \varepsilon^\circ) \rightarrow \varepsilon^\circ$
 $(\varepsilon \delta \gamma \rightarrow \alpha^\circ \beta^\circ \gamma^\circ \delta^\circ \varepsilon^\circ) \rightarrow \varepsilon^\circ$
 $(\alpha^\circ \beta^\circ \rightarrow \delta \gamma \beta \alpha) \rightarrow \varepsilon \delta \gamma \beta \alpha$
 $(\delta \gamma \beta \alpha \rightarrow \beta \alpha) \rightarrow \varepsilon \delta \gamma \beta \alpha$
 $(\beta \alpha \rightarrow \delta \gamma) \rightarrow \varepsilon \delta \gamma$
 $(\gamma^\circ \delta^\circ \rightarrow \alpha^\circ \beta^\circ) \rightarrow \varepsilon \delta \gamma$
 $(\beta \alpha \rightarrow \varepsilon \delta \gamma) \rightarrow \delta \gamma$
 $(\gamma^\circ \delta^\circ \varepsilon^\circ \rightarrow \alpha^\circ \beta^\circ) \rightarrow \delta \gamma$
 $(\alpha^\circ \beta^\circ \rightarrow \varepsilon \delta \gamma \beta \alpha) \rightarrow \delta \gamma \beta \alpha$
 $(\alpha^\circ \beta^\circ \gamma^\circ \delta^\circ \varepsilon^\circ \rightarrow \beta \alpha) \rightarrow \delta \gamma \beta \alpha$
 $(\alpha^\circ \beta^\circ \gamma^\circ \delta^\circ \rightarrow \varepsilon \delta \gamma \beta \alpha) \rightarrow \beta \alpha$
 $(\alpha^\circ \beta^\circ \gamma^\circ \delta^\circ \varepsilon^\circ \rightarrow \delta \gamma \beta \alpha) \rightarrow \beta \alpha$
 $(\delta \gamma \beta \alpha \rightarrow \gamma^\circ \delta^\circ) \rightarrow \varepsilon$
 $(\delta \gamma \rightarrow \alpha^\circ \beta^\circ \gamma^\circ \delta^\circ) \rightarrow \varepsilon$

2.10. $R = (0, 000, 001, 012)$

$(\varepsilon \delta \rightarrow \varepsilon^\circ) \rightarrow \alpha^\circ \beta^\circ \gamma^\circ \delta^\circ \varepsilon^\circ$
 $(\varepsilon \rightarrow \delta^\circ \varepsilon^\circ) \rightarrow \alpha^\circ \beta^\circ \gamma^\circ \delta^\circ \varepsilon^\circ$
 $(\delta \rightarrow \varepsilon) \rightarrow \alpha^\circ \beta^\circ \gamma^\circ \delta^\circ$

$$\begin{aligned}
& (\varepsilon^\circ \rightarrow \delta^\circ) \rightarrow \alpha^\circ \beta^\circ \gamma^\circ \delta^\circ \\
& (\delta^\circ \rightarrow \varepsilon \delta) \rightarrow \alpha^\circ \beta^\circ \gamma^\circ \\
& (\delta^\circ \varepsilon^\circ \rightarrow \delta^\circ) \rightarrow \alpha^\circ \beta^\circ \gamma^\circ \\
& (\varepsilon \delta \gamma \beta \alpha \rightarrow \varepsilon^\circ) \rightarrow \delta^\circ \varepsilon^\circ \\
& (\varepsilon \rightarrow \alpha^\circ \beta^\circ \gamma^\circ \delta^\circ \varepsilon^\circ) \rightarrow \delta^\circ \varepsilon^\circ \\
& (\delta \gamma \beta \alpha \rightarrow \varepsilon) \rightarrow \delta^\circ \\
& (\varepsilon^\circ \rightarrow \alpha^\circ \beta^\circ \gamma^\circ \delta^\circ) \rightarrow \delta^\circ \\
& (\varepsilon \delta \gamma \beta \alpha \rightarrow \delta^\circ \varepsilon^\circ) \rightarrow \varepsilon^\circ \\
& (\varepsilon \delta \rightarrow \alpha^\circ \beta^\circ \gamma^\circ \delta^\circ \varepsilon^\circ) \rightarrow \varepsilon^\circ \\
& (\alpha^\circ \beta^\circ \gamma^\circ \rightarrow \delta \gamma \beta \alpha) \rightarrow \varepsilon \delta \gamma \beta \alpha \\
& (\alpha^\circ \beta^\circ \gamma^\circ \delta^\circ \rightarrow \gamma \beta \alpha) \rightarrow \varepsilon \delta \gamma \beta \alpha \\
& (\alpha^\circ \beta^\circ \gamma^\circ \rightarrow \varepsilon \delta \gamma \beta \alpha) \rightarrow \delta \gamma \beta \alpha \\
& (\alpha^\circ \beta^\circ \gamma^\circ \delta^\circ \varepsilon^\circ \rightarrow \gamma \beta \alpha) \rightarrow \delta \gamma \beta \alpha \\
& (\alpha^\circ \beta^\circ \gamma^\circ \delta^\circ \rightarrow \varepsilon \delta \gamma \beta \alpha) \rightarrow \gamma \beta \alpha \\
& (\alpha^\circ \beta^\circ \gamma^\circ \delta^\circ \varepsilon^\circ \rightarrow \delta \gamma \beta \alpha) \rightarrow \gamma \beta \alpha \\
& (\gamma \beta \alpha \rightarrow \delta) \rightarrow \varepsilon \delta \\
& (\delta^\circ \rightarrow \alpha^\circ \beta^\circ \gamma^\circ) \rightarrow \varepsilon \delta \\
& (\gamma \beta \alpha \rightarrow \varepsilon \delta) \rightarrow \delta \\
& (\delta^\circ \varepsilon^\circ \rightarrow \alpha^\circ \beta^\circ \gamma^\circ) \rightarrow \delta \\
& (\delta \gamma \beta \alpha \rightarrow \delta^\circ) \rightarrow \varepsilon \\
& (\delta \rightarrow \alpha^\circ \beta^\circ \gamma^\circ \delta^\circ) \rightarrow \varepsilon
\end{aligned}$$

2.11. $R = (00, 01, 000, 001)$

$$\begin{aligned}
& (\delta \gamma \rightarrow \delta^\circ) \rightarrow \beta^\circ \gamma^\circ \delta^\circ \\
& (\delta \rightarrow \gamma^\circ \delta^\circ) \rightarrow \beta^\circ \gamma^\circ \delta^\circ
\end{aligned}$$

$$\begin{aligned}
(\gamma \rightarrow \delta) &\rightarrow \beta^\circ \gamma^\circ \\
(\delta^\circ \rightarrow \gamma^\circ) &\rightarrow \beta^\circ \gamma^\circ \\
(\gamma^\circ \rightarrow \delta \gamma) &\rightarrow \beta^\circ \\
(\gamma^\circ \delta^\circ \rightarrow \gamma) &\rightarrow \beta^\circ \\
(\delta \gamma \beta \rightarrow \delta^\circ) &\rightarrow \gamma^\circ \delta^\circ \\
(\delta \rightarrow \beta^\circ \gamma^\circ \delta^\circ) &\rightarrow \gamma^\circ \delta^\circ \\
(\gamma \beta \rightarrow \delta) &\rightarrow \gamma^\circ \\
(\delta^\circ \rightarrow \beta^\circ \gamma^\circ) &\rightarrow \gamma^\circ \\
(\delta \gamma \beta \rightarrow \gamma^\circ \delta^\circ) &\rightarrow \delta^\circ \\
(\delta \gamma \rightarrow \beta^\circ \gamma^\circ \delta^\circ) &\rightarrow \delta^\circ \\
(\beta^\circ \rightarrow \gamma \beta) &\rightarrow \delta \gamma \beta \\
(\beta^\circ \gamma^\circ \rightarrow \beta) &\rightarrow \delta \gamma \beta \\
(\beta \rightarrow \gamma) &\rightarrow \delta \gamma \\
(\gamma^\circ \rightarrow \beta^\circ) &\rightarrow \delta \gamma \\
(\beta^\circ \rightarrow \delta \gamma \beta) &\rightarrow \gamma \beta \\
(\beta^\circ \gamma^\circ \delta^\circ \rightarrow \beta) &\rightarrow \gamma \beta \\
(\gamma \beta \rightarrow \gamma^\circ) &\rightarrow \delta \\
(\gamma \rightarrow \beta^\circ \gamma^\circ) &\rightarrow \delta \\
(\beta \rightarrow \delta \gamma) &\rightarrow \gamma \\
(\gamma^\circ \delta^\circ \rightarrow \beta^\circ) &\rightarrow \gamma \\
(\beta^\circ \gamma^\circ \rightarrow \delta \gamma \beta) &\rightarrow \beta \\
(\beta^\circ \gamma^\circ \delta^\circ \rightarrow \gamma \beta) &\rightarrow \beta
\end{aligned}$$

2.12. R = (00, 01, 000, 012)

$$(\varepsilon\delta\gamma \rightarrow \delta^\circ\varepsilon^\circ) \rightarrow \beta^\circ\gamma^\circ\delta^\circ\varepsilon^\circ$$

$$(\varepsilon\delta \rightarrow \gamma^\circ\delta^\circ\varepsilon^\circ) \rightarrow \beta^\circ\gamma^\circ\delta^\circ\varepsilon^\circ$$

$$(\varepsilon\delta\gamma\beta \rightarrow \delta^\circ\varepsilon^\circ) \rightarrow \gamma^\circ\delta^\circ\varepsilon^\circ$$

$$(\varepsilon\delta \rightarrow \beta^\circ\gamma^\circ\delta^\circ\varepsilon^\circ) \rightarrow \gamma^\circ\delta^\circ\varepsilon^\circ$$

$$(\gamma \rightarrow \varepsilon\delta) \rightarrow \beta^\circ\gamma^\circ$$

$$(\delta^\circ\varepsilon^\circ \rightarrow \gamma^\circ) \rightarrow \beta^\circ\gamma^\circ$$

$$(\gamma^\circ \rightarrow \varepsilon\delta\gamma) \rightarrow \beta^\circ$$

$$(\gamma^\circ\delta^\circ\varepsilon^\circ \rightarrow \gamma) \rightarrow \beta^\circ$$

$$(\gamma\beta \rightarrow \varepsilon\delta) \rightarrow \gamma^\circ$$

$$(\delta^\circ\varepsilon^\circ \rightarrow \beta^\circ\gamma^\circ) \rightarrow \gamma^\circ$$

$$(\beta^\circ \rightarrow \gamma\beta) \rightarrow \varepsilon\delta\gamma\beta$$

$$(\beta^\circ\gamma^\circ \rightarrow \beta) \rightarrow \varepsilon\delta\gamma\beta$$

$$(\beta \rightarrow \gamma) \rightarrow \varepsilon\delta\gamma$$

$$(\gamma^\circ \rightarrow \beta^\circ) \rightarrow \varepsilon\delta\gamma$$

$$(\beta^\circ \rightarrow \varepsilon\delta\gamma\beta) \rightarrow \gamma\beta$$

$$(\beta^\circ\gamma^\circ\delta^\circ\varepsilon^\circ \rightarrow \beta) \rightarrow \gamma\beta$$

$$(\beta \rightarrow \varepsilon\delta\gamma) \rightarrow \gamma$$

$$(\gamma^\circ\delta^\circ\varepsilon^\circ \rightarrow \beta^\circ) \rightarrow \gamma$$

$$(\beta^\circ\gamma^\circ \rightarrow \varepsilon\delta\gamma\beta) \rightarrow \beta$$

$$(\beta^\circ\gamma^\circ\delta^\circ\varepsilon^\circ \rightarrow \gamma\beta) \rightarrow \beta$$

$$(\varepsilon\delta\gamma\beta \rightarrow \gamma^\circ\delta^\circ\varepsilon^\circ) \rightarrow \delta^\circ\varepsilon^\circ$$

$$(\varepsilon\delta\gamma \rightarrow \beta^\circ\gamma^\circ\delta^\circ\varepsilon^\circ) \rightarrow \delta^\circ\varepsilon^\circ$$

$$(\gamma\beta \rightarrow \gamma^\circ) \rightarrow \varepsilon\delta$$

$$(\gamma \rightarrow \beta^\circ\gamma^\circ) \rightarrow \varepsilon\delta$$

2.13. R = (00, 01, 001, 012)

$$(\varepsilon\delta\gamma \rightarrow \varepsilon^\circ) \rightarrow \beta^\circ\gamma^\circ\delta^\circ\varepsilon^\circ$$

$$(\varepsilon \rightarrow \gamma^\circ\delta^\circ\varepsilon^\circ) \rightarrow \beta^\circ\gamma^\circ\delta^\circ\varepsilon^\circ$$

$$(\delta\gamma \rightarrow \varepsilon) \rightarrow \beta^\circ\gamma^\circ\delta^\circ$$

$$(\varepsilon^\circ \rightarrow \gamma^\circ\delta^\circ) \rightarrow \beta^\circ\gamma^\circ\delta^\circ$$

$$(\varepsilon\delta\gamma\beta \rightarrow \varepsilon^\circ) \rightarrow \gamma^\circ\delta^\circ\varepsilon^\circ$$

$$(\varepsilon \rightarrow \beta^\circ\gamma^\circ\delta^\circ\varepsilon^\circ) \rightarrow \gamma^\circ\delta^\circ\varepsilon^\circ$$

$$(\delta\gamma\beta \rightarrow \varepsilon) \rightarrow \gamma^\circ\delta^\circ$$

$$(\varepsilon^\circ \rightarrow \beta^\circ\gamma^\circ\delta^\circ) \rightarrow \gamma^\circ\delta^\circ$$

$$(\gamma^\circ\delta^\circ \rightarrow \varepsilon\delta\gamma) \rightarrow \beta^\circ$$

$$(\gamma^\circ\delta^\circ\varepsilon^\circ \rightarrow \delta\gamma) \rightarrow \beta^\circ$$

$$(\varepsilon\delta\gamma\beta \rightarrow \gamma^\circ\delta^\circ\varepsilon^\circ) \rightarrow \varepsilon^\circ$$

$$(\varepsilon\delta\gamma \rightarrow \beta^\circ\gamma^\circ\delta^\circ\varepsilon^\circ) \rightarrow \varepsilon^\circ$$

$$(\beta^\circ \rightarrow \delta\gamma\beta) \rightarrow \varepsilon\delta\gamma\beta$$

$$(\beta^\circ\gamma^\circ\delta^\circ \rightarrow \beta) \rightarrow \varepsilon\delta\gamma\beta$$

$$(\beta \rightarrow \delta\gamma) \rightarrow \varepsilon\delta\gamma$$

$$(\gamma^\circ\delta^\circ \rightarrow \beta^\circ) \rightarrow \varepsilon\delta\gamma$$

$$(\beta^\circ \rightarrow \varepsilon\delta\gamma\beta) \rightarrow \delta\gamma\beta$$

$$(\beta^\circ\gamma^\circ\delta^\circ\varepsilon^\circ \rightarrow \beta) \rightarrow \delta\gamma\beta$$

$$(\beta \rightarrow \varepsilon\delta\gamma) \rightarrow \delta\gamma$$

$$(\gamma^\circ\delta^\circ\varepsilon^\circ \rightarrow \beta^\circ) \rightarrow \delta\gamma$$

$$(\beta^\circ\gamma^\circ\delta^\circ \rightarrow \varepsilon\delta\gamma\beta) \rightarrow \beta$$

$$(\beta^\circ\gamma^\circ\delta^\circ\varepsilon^\circ \rightarrow \delta\gamma\beta) \rightarrow \beta$$

$$(\delta\gamma\beta \rightarrow \gamma^\circ\delta^\circ) \rightarrow \varepsilon$$

$$(\delta\gamma \rightarrow \delta\gamma\beta) \rightarrow \varepsilon$$

$$2.14. R = (00, 000, 001, 012)$$

$$(\varepsilon\delta \rightarrow \varepsilon^\circ) \rightarrow \beta^\circ\gamma^\circ\delta^\circ\varepsilon^\circ$$

$$(\varepsilon \rightarrow \delta^\circ\varepsilon^\circ) \rightarrow \beta^\circ\gamma^\circ\delta^\circ\varepsilon^\circ$$

$$(\delta \rightarrow \varepsilon) \rightarrow \beta^\circ\gamma^\circ\delta^\circ$$

$$(\varepsilon^\circ \rightarrow \delta^\circ) \rightarrow \beta^\circ\gamma^\circ\delta^\circ$$

$$(\delta^\circ\varepsilon^\circ \rightarrow \delta) \rightarrow \beta^\circ\gamma^\circ$$

$$(\delta^\circ \rightarrow \varepsilon\delta) \rightarrow \beta^\circ\gamma^\circ$$

$$(\varepsilon\delta\gamma\beta \rightarrow \delta^\circ\varepsilon^\circ) \rightarrow \varepsilon^\circ$$

$$(\varepsilon\delta \rightarrow \beta^\circ\gamma^\circ\delta^\circ\varepsilon^\circ) \rightarrow \varepsilon^\circ$$

$$(\delta\gamma\beta \rightarrow \varepsilon) \rightarrow \delta^\circ$$

$$(\varepsilon^\circ \rightarrow \delta\gamma\beta) \rightarrow \delta^\circ$$

$$(\varepsilon\delta\gamma\beta \rightarrow \varepsilon^\circ) \rightarrow \delta^\circ\varepsilon^\circ$$

$$(\varepsilon \rightarrow \varepsilon\delta\gamma\beta) \rightarrow \delta^\circ\varepsilon^\circ$$

$$(\beta^\circ\gamma^\circ \rightarrow \delta\gamma\beta) \rightarrow \varepsilon\delta\gamma\beta$$

$$(\beta^\circ\gamma^\circ\delta^\circ \rightarrow \gamma\beta) \rightarrow \varepsilon\delta\gamma\beta$$

$$(\beta^\circ\gamma^\circ \rightarrow \varepsilon\delta\gamma\beta) \rightarrow \delta\gamma\beta$$

$$(\beta^\circ\gamma^\circ\delta^\circ\varepsilon^\circ \rightarrow \gamma\beta) \rightarrow \delta\gamma\beta$$

$$(\beta^\circ\gamma^\circ\delta^\circ\varepsilon^\circ \rightarrow \delta\gamma\beta) \rightarrow \gamma\beta$$

$$(\beta^\circ\gamma^\circ\delta^\circ \rightarrow \varepsilon\delta\gamma\beta) \rightarrow \gamma\beta$$

$$(\gamma\beta \rightarrow \delta) \rightarrow \varepsilon\delta$$

$$(\delta^\circ \rightarrow \beta^\circ\gamma^\circ) \rightarrow \varepsilon\delta$$

$$(\gamma\beta \rightarrow \varepsilon\delta) \rightarrow \delta$$

$$(\delta^\circ\varepsilon^\circ \rightarrow \beta^\circ\gamma^\circ) \rightarrow \delta$$

$$(\delta\gamma\beta \rightarrow \delta^\circ) \rightarrow \varepsilon$$

$$(\delta \rightarrow \beta^\circ\gamma^\circ\delta^\circ) \rightarrow \varepsilon$$

$$2.15. R = (01, 000, 001, 012)$$

$$(\varepsilon\delta \rightarrow \varepsilon^\circ) \rightarrow \gamma^\circ\delta^\circ\varepsilon^\circ$$

$$(\varepsilon \rightarrow \delta^\circ\varepsilon^\circ) \rightarrow \gamma^\circ\delta^\circ\varepsilon^\circ$$

$$(\delta \rightarrow \varepsilon) \rightarrow \gamma^\circ\delta^\circ$$

$$(\varepsilon^\circ \rightarrow \delta^\circ) \rightarrow \gamma^\circ\delta^\circ$$

$$(\varepsilon\delta\gamma \rightarrow \varepsilon^\circ) \rightarrow \delta^\circ\varepsilon^\circ$$

$$(\varepsilon \rightarrow \gamma^\circ\delta^\circ\varepsilon^\circ) \rightarrow \delta^\circ\varepsilon^\circ$$

$$(\delta^\circ \rightarrow \varepsilon\delta) \rightarrow \gamma^\circ$$

$$(\delta^\circ\varepsilon^\circ \rightarrow \delta) \rightarrow \gamma^\circ$$

$$(\delta\gamma \rightarrow \varepsilon) \rightarrow \delta^\circ$$

$$(\varepsilon^\circ \rightarrow \gamma^\circ\delta^\circ) \rightarrow \delta^\circ$$

$$(\varepsilon\delta\gamma \rightarrow \delta^\circ\varepsilon^\circ) \rightarrow \varepsilon^\circ$$

$$(\varepsilon \rightarrow \gamma^\circ\delta^\circ\varepsilon^\circ) \rightarrow \varepsilon^\circ$$

$$(\gamma^\circ \rightarrow \varepsilon\delta\gamma) \rightarrow \varepsilon\delta\gamma$$

$$(\gamma^\circ\delta^\circ\varepsilon^\circ \rightarrow \gamma) \rightarrow \varepsilon\delta\gamma$$

$$(\gamma^\circ\delta^\circ \rightarrow \gamma) \rightarrow \delta\gamma$$

$$(\gamma^\circ \rightarrow \delta\gamma) \rightarrow \delta\gamma$$

$$(\gamma^\circ\delta^\circ \rightarrow \varepsilon\delta\gamma) \rightarrow \gamma$$

$$(\gamma^\circ\delta^\circ\varepsilon^\circ \rightarrow \delta\gamma) \rightarrow \gamma$$

$$(\gamma \rightarrow \delta) \rightarrow \varepsilon\delta$$

$$(\delta^\circ \rightarrow \gamma^\circ) \rightarrow \varepsilon\delta$$

$$(\gamma \rightarrow \varepsilon\delta) \rightarrow \delta$$

$(\delta^\circ \varepsilon^\circ \rightarrow \gamma^\circ) \rightarrow \delta$

$(\delta \gamma \rightarrow \delta^\circ) \rightarrow \varepsilon$

$(\delta \rightarrow \gamma^\circ \delta^\circ) \rightarrow \varepsilon$

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20.6.2019